Numerical modelling of anisotropic diffusion in Earth's radiation belts

Nourallah DAHMEN[1], Vincent MAGET[1], François ROGIER[2]

(1) ONERA - Toulouse/DPHY - 2, Avenue Edouard Belin, 31400 Toulouse, France

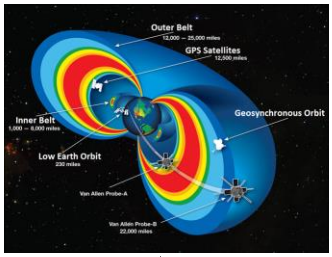
(2) ONERA - Toulouse/DTIS - 2, Avenue Edouard Belin, 31400 Toulouse, France

Space environment & Radiation belts

- Near-Earth environment : Charged particles interact with Earth's magnetosphere, EM waves, solar and cosmic rays.

- Radiation belts : Energetic p+ and e- are **pseudo-trapped** in the magnetic field.

They describe a ***pseudo-periodic*** *motion.*



Credit : NASA

Salammbô 3D : Numerical code for Radiation belts dynamics

- 3D model (E,y,L and t) : Developed since 90's by **DPHY/ONERA**. Empirical entries, space environment model (Dipolar tilted Earth's magnetic field, atmospheric particle densities, Sun-Earth interaction…).

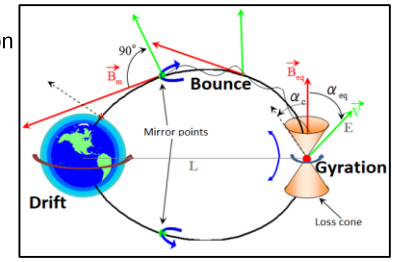
- Discretization : **Finite difference** in space and **Euler explicit** in time. Space grid refinement : 133x49x133 – Time step Δt =1s

Physics and modeling

- Adiabatic motion of charged particles : Trapped particles motion is decomposed into 3 elementary motions **Gyration**, **Bounce** and **Drift** related to 3 conserved quantities.

Adiabatic invariants theory

- Fokker Planck Equation : Hamiltonian formulation and statistical approach (Boltzmann equation)



Decomposed motion of charged particles and new introduced variables.

$$\frac{\partial f}{\partial t}+\sum_{i=2}^{3}\frac{dJ_i}{dt}.\frac{\partial f}{\partial J_i}+\sum_{i=1}^{3}\frac{d\varphi_i}{dt}.\frac{\partial f}{\partial \varphi_i}=-\sum_{i=1}^{3}\frac{\partial}{\partial J_i}(D_{J_i}.f)+\frac{1}{2}\sum_{i=1}^{3}\sum_{j=1}^{3}\frac{\partial^2}{\partial J_i \partial J_j}(D_{J_iJ_j}.f)+\left(\frac{\partial f}{\partial t}\right)_c$$

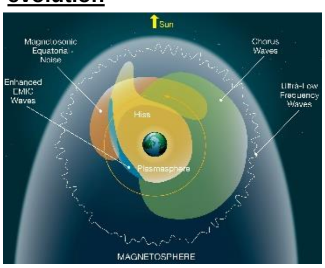
f : distribution function ($Mev^{-3}s^{-3}$) , ($J1,J2,J3,\varphi_1,\varphi_2,\varphi_3$) : space phase

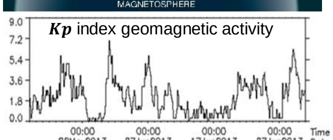
- Space reduction and change of variables :

$$\frac{\partial f}{\partial t}=\frac{1}{G}\frac{\partial}{\partial y_{E,L}}\left(G(D_{yy}\frac{\partial f}{dy}_{E,L}+D_{yE}\frac{\partial f}{dE}_{y,L})\right)+\frac{1}{G}\frac{\partial}{\partial E_{y,L}}\left(G(D_{EE}\frac{\partial f}{dE}_{y,L}+D_{Ey}\frac{\partial f}{dy}_{E,L})\right)+L^2\frac{\partial}{\partial L_{\mu,J}}\left(\frac{D_{LL}}{L^2}\frac{\partial f}{dL_{\mu,J}}\right)$$

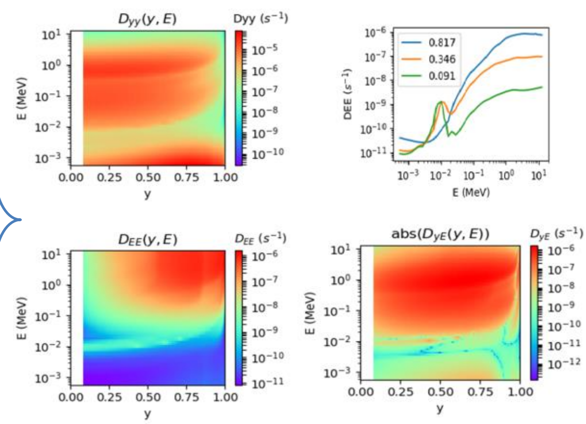
The challenges of the numerical resolution : Anisotropy and strong gradients

- Multiscale medium and volatile evolution





- Inhomogeneous and highly anisotropic tensor

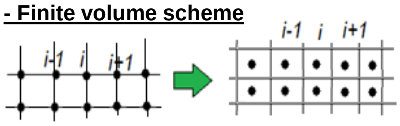


Brutal ***physical evolution*** *⇒ huge* ***constraints*** *on* ***numerical resolution***

- **Coefficient inhomogeneity** → **Accuracy**
- **Huge anisotropy** → **Loss of solution physicality**
- **Stiff evolution** → **Stability**

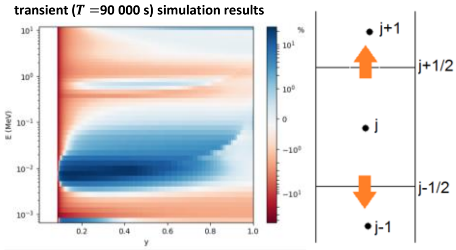
Dedicated finite volume scheme

- Finite volume scheme



Numerical flux balance *with flux* ***conservation*** *at edge*

Finite volume VS Finite difference 2D y,E (133 x 49) for L =4.49 transient (T =90 000 s) simulation results



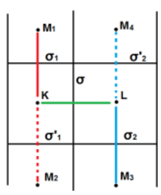
Significant impact on the gradient estimation and accuracy

- Non-linear scheme

Physical solution = preserve **positivity / Min-Max principles**

Num. Flux ponderation :

$$F_{K,\sigma}=\mu_1 F_1+\mu_2 F_2$$



$\mu_1(f)$ and $\mu_2(f)$ fixed to achieve a monotone (NL-TPFA [1]) or MMP structure (NL-MPFA [1])

2D y,E (133 x 49) for L =4.49 transient (T =90 000 s) simulation results

Scheme	$\Delta t=100$s		$\Delta t=1000$s	
	N_{it}	T_{simu}	N_{it}	T_{simu}
NLTPFA	7,99	4978 s	20,98	21526 s
NLMPFA	19,69	21928 s	33,18	4272 s

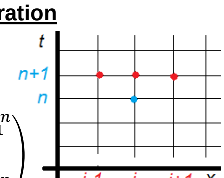
Need to tackle simulation cost challenge

(1) Droniou J. Finite volume schemes for diffusion equations: introduction to and review of modern methods. Mathematical Models and Methods in Applied Sciences. 2014;24(8):1575 - 1619.

- Implicit time integration

$$\frac{\partial f}{\partial t}\cong\frac{f^{n+1}-f^n}{\Delta t}=L^{n+1}(f^{n+1})$$

$$F^n=B\,F^{n+1}, F^n=\begin{pmatrix}f_1^n\\ \vdots\\ f_N^n\end{pmatrix}$$



2D y,E (34 x 25) for L =4.49 transient (T =10 000 s) simulation results

Scheme	Explicit $\Delta t=0.05s$	Implicit $\Delta t=100s$
T_{simu}	142.55 s	0.23 s

$$0{,}998<ratio\frac{PSD_{implicit}}{PSD_{explicit}}<1{,}005$$

Relaxation of the time step choice constraint

Next ***: Splitting resolution*** → **2D** (y,E) + **1D** (L)