Galactic binaries with LISA

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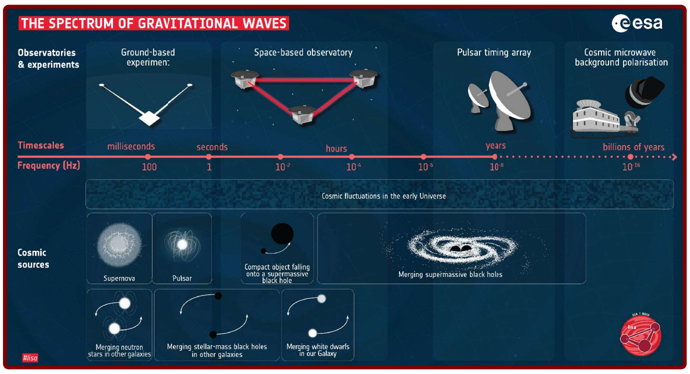
Laser Interferometer Space Antenna (LISA)

LISA is an upcoming *space-based gravitational wave observatory*. As ESA's third large-class mission, LISA is planned to launch in 2035 with a mission duration of 4–5 years. The detector setup will consist of three separate spacecraft in triangular formation 2.5 million km apart, trailing the Earth in heliocentric orbit. The spacecraft will be interconnected by lasers which can detect passing gravitational waves via time-delay interferometry.

Current ground-based instruments can detect the final moments of the merger (i.e. the plunge) of stellar-sized objects like black-hole and neutron-star pairs. Space-based instruments have the potential to peer into a range of much lower frequencies, accessing a rich variety of sources, from supermassive black-holes in galactic centres to primordial events (see Fig.). Among other sources, LISA will continuously observe the gravitational waves emitted by inspiraling white-dwarf and neutron-star binaries within the Milky Way (i.e. galactic binaries). The data acquisition can cover years for each system, with tens of thousands of binaries monitored simultaneously.

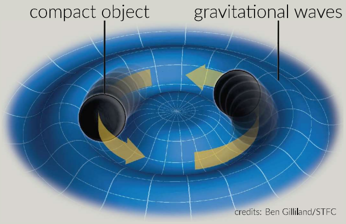
These observations will enable the characterisation of the source systems and our understanding of their internal physics and state of matter. Detection is based on matched-filtering, and shall require templates which remain accurate over thousands of orbits. This suggests further refinement of dynamical models considering a variety of physical effects, including spin and magnetic field interaction, tidal forces, and radiation-reaction effects.



Gravitational waves

Gravitational waves (GWs) are minuscule ripples in spacetime produced from the acceleration of massive bodies. Predicted back in the early days of General Relativity, they were finally observed directly in 2015 by the LIGO-VIRGO observatories.

As they travel, GWs deform spacetime by alternately stretching and compressing it. They can be polarised into two independent 'modes' $h_+(t)$ and $h_\times(t)$.



The GWs emitted by a binary can be computed from the quadrupole formula:

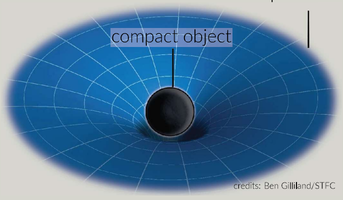
$$h_{ij}(t, \boldsymbol{x}) = \frac{2G}{c^4 x} \frac{\mathrm{d}^2}{\mathrm{d}t^2} Q_{ij}(t) \qquad (1)$$

where $Q_{ij}(t)$ is the mass quadrupole moment, which is a function of the orbital trajectory.

Thus, an analytical description of the trajectory of the source is required for modelling the gravitational waveform. This waveform may conceivably depend on many parameters of each body, such as masses, spins, magnetic and tidal properties, etc.

Post-Newtonian expansions

The Einstein field equations describe how matter and energy influence the curvature of spacetime, which in turn dictates the motion of bodies. Despite an elegant formulation, scarcely any exact solutions are known. Most practical scenarios, such as merging black holes or other compact systems, require the use of numerical or approximation methods.



Among analytical methods, post-Newtonian (PN) expansion schemes excel at describing 'weak-field' binary orbits, such as inspiraling compact binaries. The approach involves expanding the field equations perturbatively in powers of $\varepsilon \sim v/c$ and solving for the curvature (or rather the 'spacetime metric'). By doing so, one is left with **equations of motion** for the bodies [3], occasionally in the form of a Lagrangian or a Hamiltonian [8], resembling those of Newtonian gravity with additional relativistic corrections.

While post-Newtonian expansions have been progressively reaching higher orders, the development of tools to analytically **solve** the resulting equations remains behind, with ad-hoc methods frequently restricted to secular or quasi-circular approximations. The quasi-Keplerian parametrisation (see [3]) has been sucessfully determined up to 4PN order to describe the orbits of non-spinning binaries. However, a systematic extension of this approach to broader perturbative situations including spinning or dissipative systems remains a challenge.

Orbital motion

The orbital dynamics of a purely gravitational binary system can be described by the post-Newtonian Arnowitt–Deser–Misner (ADM) formalism. For a non-spinning point-mass binary, the conservative Hamiltonian is:

$$\mathcal{H}(\boldsymbol{r}, \boldsymbol{p}) = \mathcal{H}_0(\boldsymbol{r}, \boldsymbol{p}) + \varepsilon^2 \mathcal{H}_1(\boldsymbol{r}, \boldsymbol{p}) + \varepsilon^4 \mathcal{H}_2(\boldsymbol{r}, \boldsymbol{p}) + \dots, \qquad (2)$$

where $(\boldsymbol{r}, \boldsymbol{p})$ are the binary separation and conjugate momentum seen from the centre-of-mass frame, and the leading term $\mathcal{H}_0$ is the Hamiltonian for Newtonian gravity. Any further dissipative or non-gravitational interactions will show up as additional perturbations.

In general, the above Hamiltonian is not directly integrable, due to the presence of the non-linear perturbation terms $\varepsilon^2\mathcal{H}_1$, $\varepsilon^4\mathcal{H}_2$, etc. In [1], we propose incorporating the Lie series approach into the post-Newtonian context. This technique, described below, provides a systematic framework for computing the orbital trajectories, capturing both long-term (secular) and short-term dynamics, arbitrary eccentricities, and can be applied to systems with a diverse range of perturbations.

The Lie series method

The approach involves finding a near-identity canonical coordinate transformation $\mathcal{T}_g$, which maps the original Hamiltonian $\mathcal{H}$ into a new Hamiltonian $\mathcal{H}^* = \mathcal{T}_g(\mathcal{H})$. The mapping is carefully chosen such that the non-integrable terms in $\mathcal{H}$ are deferred to high PN orders in $\mathcal{H}^*$, where they can be formally neglected. At this point, the dynamics of $\mathcal{H}^*$ can be easily extracted, giving the secular orbital motion $(\boldsymbol{r}^*, \boldsymbol{p}^*)$. Finally, the process is reversed by applying $\mathcal{T}_g$ to the secular coordinates, recovering the complete dynamics of the system.

$$\begin{array}{ccc} \mathcal{H} = \sum_{\ell=0}^{K} \varepsilon^{2\ell} \mathcal{H}_\ell & \xrightarrow{\mathcal{T}_g} & \mathcal{H}^* = \mathcal{T}_g(\mathcal{H}) \\ \downarrow & & \downarrow \\ \boldsymbol{r}(t),\, \boldsymbol{p}(t) & \xleftarrow{\mathcal{T}_g} & \boldsymbol{r}^*(t),\, \boldsymbol{p}^*(t) \end{array}$$

The core challenge of the approach lies in determining the generator of the transformation, which parametrically encodes $\mathcal{T}_g$ as:

$$\mathcal{T}_g(x) = x + \varepsilon^2\{x, g\} + \frac{\varepsilon^4}{2}\{\{x, g\}, g\} + \dots,$$

where $\{\cdot, \cdot\}$ is the Poisson bracket. To find the generator, we progressively solve equations of the form:

$$\{g_\ell, \mathcal{H}_0\} = F_\ell(\boldsymbol{r}, \boldsymbol{p}) - \mathcal{H}^*_\ell, \quad \ell = 1, 2, \dots, \qquad (3)$$

where $F_\ell(\boldsymbol{r}, \boldsymbol{p})$ are expressions derived from the original Hamiltonian (2). The unknowns g_ℓ and $\mathcal{H}^*_\ell$ correspond to expansion terms of g and $\mathcal{H}^*$ in powers of ε^2. In [1], we derive generator solutions for (3) for typical Hamiltonian terms in (2), which include the *local conservative* ADM sector but also other rotation-invariant perturbations. The framework can be naturally extended to include non-conservative contributions such as radiation-reaction terms (Aykroyd et al., in prep.), by using the variable-doubling formalism [5].

Magnetic interactions

Isolated white dwarfs and neutron stars can exhibit external magnetic fields as strong as 10^9 and 10^{15} Gauss, respectively. However, whether such strongly magnetic degenerate stars are commonly found in binary systems remains unclear. Clarifying this subtlety could provide crucial information on the stability and formation mechanisms of magnetic fields in these binaries.

Recent studies show that magnetic interactions in galactic binaries create distinct signatures in GWs that should be detectable by LISA [4, 6]. Analyzing these signals can reveal the magnetic properties of the systems, providing valuable data for population models and insights into the magnetic fields of degenerate stars.

Magnetic model

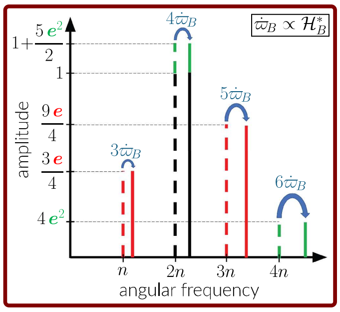
At leading order, the perturbing magnetic interaction can be encoded by a non-relativistic term:

$$\mathcal{H}_B = \frac{\mu_0}{4\pi r^3}\big(\boldsymbol{\mu}_1 \cdot \boldsymbol{\mu}_2 - 3(\boldsymbol{\mu}_1 \cdot \boldsymbol{r})(\boldsymbol{\mu}_2 \cdot \boldsymbol{r})\big). \qquad (4)$$

where the dipole moments $\boldsymbol{\mu}_{1,2}$ are assumed to be aligned with the spins of the stars. We show [2] that given enough time, the magnetic interaction will drive the dipoles into a configuration where they are anti-aligned and perpendicular to the plane of the orbit.

Magnetic signature in GWs

In this configuration, we can compute the orbital motion and demonstrate [7] that magnetism will manifest as a frequency shift in the GW mode harmonics proportional to the magnetic interaction energy (4).



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