Decoding Optical Aberrations of Low-Resolution Instruments from PSFs: Machine Learning and Zernike Polynomials Perspectives

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INTRODUCTION

The accurate modeling of the Point Spread Function (PSF) is essential to understand the instrumental impact on images taken by optical systems. As a key indicator of an instrument's optical quality, it reveals the nature of its optical aberrations which are fundamental in evaluating image quality and systematics. Knowledge of this instrumental response plays a crucial role in applications such as image deconvolution, where it enables the extraction of ideal images from observed data. While deconvolving images using the PSF, we improve the image clarity and allow the extraction of finer details, and such an improvement on images global quality can advance our comprehension of physical phenomena.

In this study, our objective is to develop a machine learning model that can establish a connection between Zernike coefficients, which define the Wavefront Error (WFE) of optically deformed systems, and the related PSF images. Resulting from the combination of neural networks and Zernike Polynomials, a brand-new model was created : ZerNet.

METHODS

Zernike Polynomials and WFE

When describing the Wavefront Error, Zernike Polynomials stand as a powerful tool for representing a wide range of optical aberrations, the lowest orders commonly acknowledged by scientists are represented below.

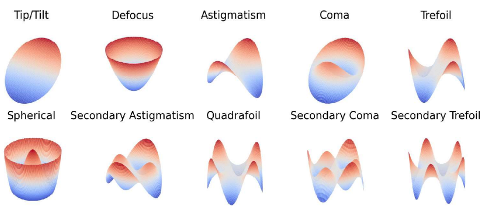

Fig. 1 Visualization of Zernike Polynomials and optical aberrations

We may describe WFE, as a linear combination of infinite order [1] :

$$\Phi(\rho,\theta) = \sum_{j=1}^{\infty} \alpha_j Z_j(\rho,\theta)$$

However, in real-world situations, optical systems are tuned to reduce higher-order aberrations, thereby limiting Noll's index j to N_{orders}. This allows us to express the maximum values of each radial order (in nanometers of optical path difference) as a finite WFE budget for the Zernike coefficients α_j in our simulations :

$$B = [0,100,50,36,18,9]$$

PSF simulation

The Python module `poppy` [2] allowed to model a telescope with a Newtonian shape, consisting of two mirrors, a primary and a secondary supported by three struts.

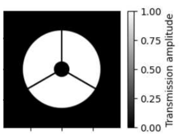

Fig. 2 Designated pupil

Such a configuration is commonly used, particularly in astronomy, while remaining basic allowing to be customized and adapted.

Unfortunately, one main issue stands out while using machine learning algorithms : it requires extensive samples of data.

`poppy` stood out as the perfect tool for our study because it includes a ZernikeWFE class implementation and parallelized PSF calculations, which significantly reduces the typically large amount of time needed to generate PSFs.

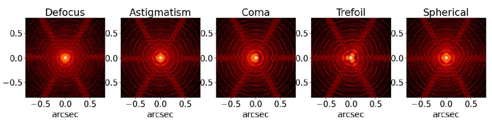

Fig. 3 Aberrated PSFs simulated at 0.2 µm

As a result, it allows the computation of PSFs shown in the previous plot, where optical aberrations mainly play a role in the center part of the PSF.

Fraunhofer approximation

While `poppy` allows to compute PSFs from Fresnel and Fraunhofer regions, choice has been made to take advantage of Fraunhofer diffraction region which implies both far field and paraxial hypothesis, very common in astronomy.

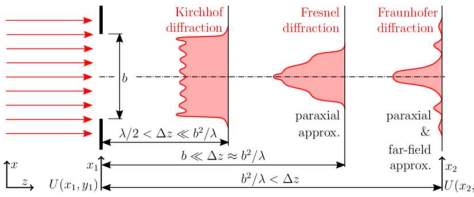

Fig. 4 Diffraction regions after aperture / Credit : Figure from Störkle [3]

After selecting a wavelengths range λ between 0.2 µm and 1 µm, we made use of encircled energy (EE) as estimator of our PSF loss while cropping it to the region of interest we need.

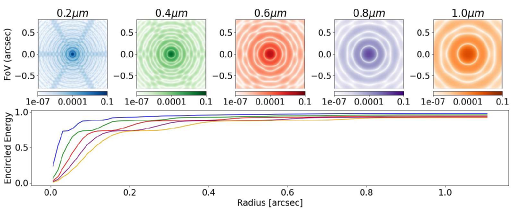

Fig. 5 PSFs and EEs for λ between 0.2 µm and 1 µm

Fortunately, for any value of λ, our chosen region of interest (ROI) was big enough to contain more than 98% of the PSF.

ZerNet model

To link the images and their corresponding coefficients, we made use of Zernet. This study's model is built on the Inception model's premise and converts a $2D$ tensor as the input and a $1D$ tensor of N_{orders} values as the output.

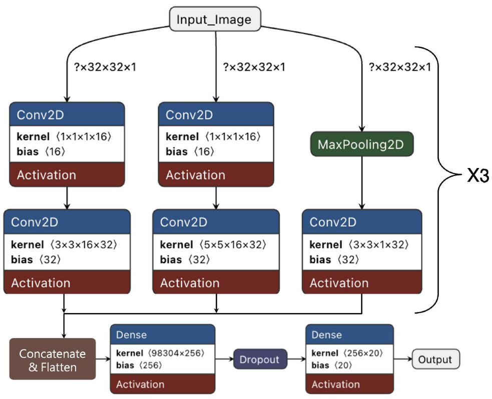

Fig. 6 ZerNet model architecture

RESULTS

ZerNet Performance with 1µm PSFs

The following figure features an upper plot comparing the Zernike coefficient of the model with the reconstructed ZerNet predictions, colors corresponding to different orders. The lower plot shows the error over a range of real coefficients values.

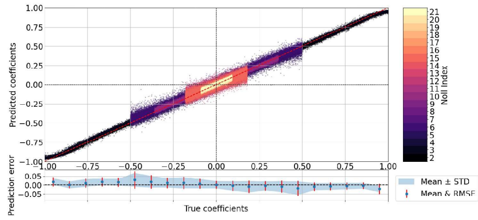

Fig. 7 Overview of Zernet's performance for coefficients estimation

The PSF reconstruction error is crucial to the propagation of uncertainty in scientific applications, and it must be minimized to limit systematics. In this instance, the error ϵ resulting from coefficients' estimation until order j_{max} was measured using the Frobenius norm [4]:

$$\epsilon_{j_{max}} = \left\| \left(PSF_{True} - PSF(\alpha_{:j_{max}}) \right) \right\|_F$$

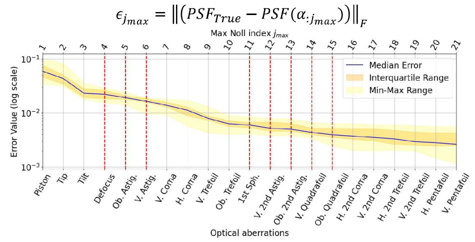

Fig. 8 Error Analysis Between True and Reconstructed PSFs

The previous plot showcases, through $\epsilon_{j_{max}}$ computation, the ability of ZerNet to precisely assess optical system aberrations. Even though min-max range appears wide and variable, the continuous decrease of median error confirms that using the predictions of several orders is constructive.

DISCUSSIONS

Our novel approach demonstrated that machine algorithms can establish a relationship between PSF images and corresponding optical aberrations without prior knowledge of the WFE.

Key assumptions and Constraints :

- Approximation in Fraunhofer region
- Specific WFE budget
- Low system's resolving power

Findings :

Models failed to converge on even radial orders in radially symmetrical optical system, highlighting the advantages of radially asymmetric systems.

Future Improvements:

- Add noise to match real-case scenarios : Autoencoders & Diffusion Denoising Probabilistic Models
- Train on polychromatic PSFs
- Perform element-wise analysis : WFE description per optical element
- Integrate physics into loss functions : Physics Integrated Neural Networks

In the future, it might be applied to plenty of fields such as astronomy, microscopy, or medical imaging where the understanding of WFE might pose a substantial challenge.

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