AUTONOMOUS TIME SCALE IN A SWARM OF NANOSATELLITES

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1. Context

Problem statement

A radio array telescope built using a **swarm** of nanosatellites requires a good level of clock synchronization in an environment susceptible to **anomalies**. To achieve this, we aim to generate a stable and robust **time scale**.

- **Swarm**: A collection of technologically similar satellites cooperating to achieve a common objective.
- **Anomalies**: Onboard satellite clocks are expected to suffer from missing data, and jumps in the clocks' phases and frequencies.
- **Time scale**: A common reference time that is autonomously generated with data from an ensemble of clocks, where the stability is better than any individual clock.

Radio Astronomy

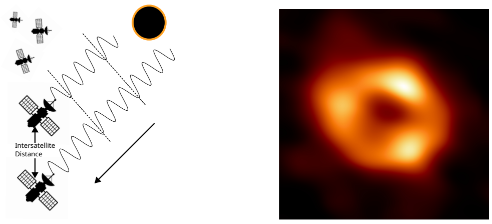


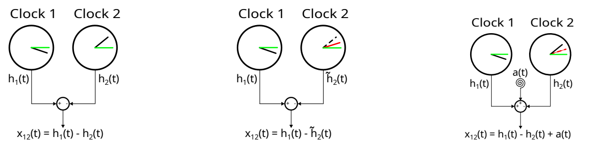
Figure 1: Illustration of potential observation scheme for space-based radio interferometry (left) and constructed image using Earth-based radio interferometry (right) Credit: EHT [1].

Space-based interferometry requires additional considerations:

- **PNT**: The estimates of inter-satellite distances are linked to the estimates of clock biases.
- **Interferometry**: A good level of synchronization is necessary to correctly combine the received signals of interest.

2. Basic Time Scale Equation

Each clock must know its time relative to the more stable timescale. In practice, we can only measure the difference in phase between pairs of clocks:



(a) Normal measurement. **(b)** Phase jump anomaly. **(c)** Measurement anomaly.

Figure 2: Measuring clocks with different anomaly sources.

The evolution of the clock states over time is random due to internal noises inside the oscillators:

$$h_i(t) = h_i(t-\tau) + \tau f_i(t-\tau) + \frac{\tau^2}{2} d_i(t-\tau) + \varepsilon_i(t) \quad (1)$$

The objective is to estimate the offset of each clock i from the stable timescale, denoted as $x_{i,E}(t) = h_i(t) - h_E(t)$. The propagation of the timescale should be predictable, with a constant frequency

$$h_E(t) = h_E(t-\tau) + \tau f_E \quad (2)$$

All satellites communicate with one another, forming a system of $N-1$ non-redundant measurements.

$$\begin{bmatrix} x_{12} \\ x_{13} \\ \vdots \\ x_{1N} \end{bmatrix} = \begin{bmatrix} x_{1,E} - x_{2,E} \\ x_{1,E} - x_{3,E} \\ \vdots \\ x_{1,E} - x_{N,E} \end{bmatrix}, \begin{bmatrix} x_{21} \\ x_{23} \\ \vdots \\ x_{2N} \end{bmatrix} = \begin{bmatrix} x_{2,E} - x_{1,E} \\ x_{2,E} - x_{3,E} \\ \vdots \\ x_{2,E} - x_{N,E} \end{bmatrix}, \dots, \begin{bmatrix} x_{N1} \\ x_{N2} \\ \vdots \\ x_{N(N-1)} \end{bmatrix} = \begin{bmatrix} x_{N,E} - x_{1,E} \\ x_{N,E} - x_{2,E} \\ \vdots \\ x_{N,E} - x_{(N-1),E} \end{bmatrix} \quad (3)$$

The system is not sufficient to solve for each $x_{i,E}$ so a new equation is introduced by using predictions according to (2)

$$\hat{x}_{i,E}(t) = x_{i,E}(t-\tau) + \tau \hat{y}(t-\tau) \quad (4)$$

Basic Time Scale Equation (BTSE): standard method of combining the predictions and measurements with the highest weights given to most predictable clocks:

$$\boxed{x_{i,E}(t) = \sum_{j=1}^{N} w_j(t-\tau)(\hat{x}_{j,E}(t) - x_{ji}(t))} \quad (5)$$

The above weights are computed according to specific algorithms in the state-of-the-art:

- **AT1**: Applies an exponential filter on the error $|x_{i,E}(t) - \hat{x}_{i,E}(t)|$. Weights are the normalized inverse of the filtered error [2].
- **KF**: Kalman Filter gain matrix applies weights simultaneously to phase, frequency, and drift estimates [3].
- **ALGOS**: Estimation of frequency and frequency variance using a window of past data. Weights are the inverses of the frequency variances [2].

The contribution of this thesis is to use the principles of the robust **Maximum Likelihood Estimator (MLE)** to determine clock weights. This is applied to both phase and frequency estimation, aiming to be robust to both phase jump and frequency jump anomalies.

3. Autonomous Timescale using Student's T-distribution

The Student's t-distribution models the likelihood of both normal and abnormal data. Consider observations of a t-distributed random variable $z_j \sim T(\mu, \sigma^2, \nu)$.

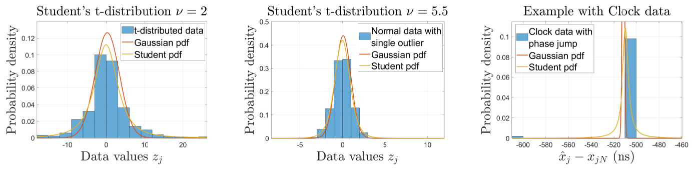


Figure 3: Distributions of t-distributed test data (left), test data corrupted by a single outlier (middle), and simulated clock data with an internal anomaly (right).

To obtain the MLE for the mean of the Student's t-distribution we can use an iterative Expectation Maximization algorithm [4].

Expectation Maximization for a Robust MLE: convergence threshold S

while $\epsilon > S$ **do**

$$u_j = \frac{\hat{\nu}_{k-1}+1}{\hat{\nu}_{k-1} + \frac{(z_j - \hat{\mu}_{k-1})^2}{\hat{\sigma}^2_{k-1}}}, \; \hat{\mu}_k = \frac{\sum_{j=1}^N u_j z_j}{\sum_{j=1}^N u_j}, \; \hat{\sigma}^2_k = \frac{\sum_{j=1}^N u_j (z_j - \hat{\mu}_{k-1})^2}{N-1},$$

$$\hat{\nu}_k = \text{root}\left(\phi\left(\frac{x}{2}\right) - \phi\left(\frac{\hat{\nu}_{k-1}+1}{2}\right) + \sum_{i=1}^N (u_i - \log(u_i) - 1)\right)$$

end while

Assumption: observations $z_j(t) = \hat{x}_j(t) - x_{ji}(t)$ follow a Student's t-distribution, with mean the clock phase offsets $\mu = x_{i,E}(t)$, scale parameter σ^2, and ν degrees of freedom.

$$\hat{x}_j(t) - x_{ji}(t) \sim T(x_{i,E}, \sigma^2, \nu) \quad (6)$$

As detailed in the EM algorithm, the phase can be estimated with a weighted average, where the weights are determined according to the **current** difference between predictions and measurements.

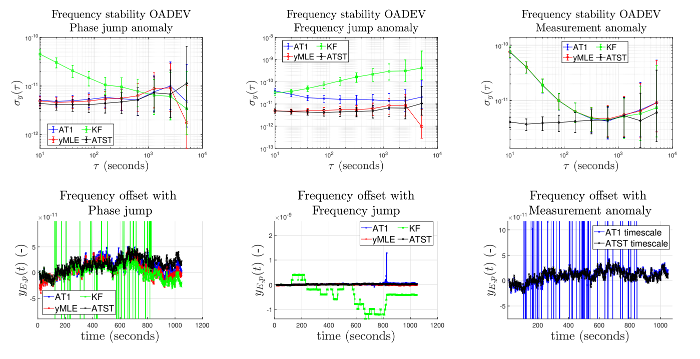
$$\boxed{x_{i,E}(t) = \frac{\sum_{j=1}^N u_j(t)(\hat{x}_{j,E}(t) - x_{ji}(t))}{\sum_{j=1}^N u_j}} \quad (7)$$

Similarly, we can assume that the frequencies of the clocks will follow a Student's t-distribution over a window of past time epochs.

3.1. Results

The resulting timescale analysis focuses on frequency stability in the form of Overlapping Allan Deviation (OADEV) and the frequency evolution of the time scale.

- **AT1**: Robustness obtained by recomputing (5) with the newly computed weights at time t before estimating frequency.
- **KF**: No inherent robustness applied, potential to apply anomaly detection methods. This example indicates the full effects of each anomaly.
- **yMLE**: Only applies the robust estimation to the frequency, then computes weights based on frequency error.
- **ATST**: Autonomous Timescale using Student's T-distribution, exploits robust estimation for both phase and frequency. This is the most robust time scale, as shown below.



4. References

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