Testing fundamental physics with the LISA space mission: Spacetime-symmetry breaking effects in galactic binaries

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Spacetime symmetries

Presently, interest in tests of foundations of **General Relativity** (GR) and the Standard Model (SM) is high, including both theory and experiment. Motivation for these studies include the possibility that some aspects of foundations of GR may be modified in a unified theory of physics that incorporates quantum gravity. In particular, suggestions that spacetime-symmetry foundations of GR, like local Lorentz symmetry, could be **broken in small but potentially detectable ways** [1]. now possible. Using gravitational-wave observations, several tests of GR have been performed, which so far has revealed no departure from known physics. Given that GR holds to very high accuracy, any spacetime-symmetry breaking in nature must be very small at the energy scales available to us, and with very little experimental guidance to direct theoretical model building, a practical approach is to search for features of the underlying theory through effective-field theory, for which we use the Standard-Model Extension (SME) [2].

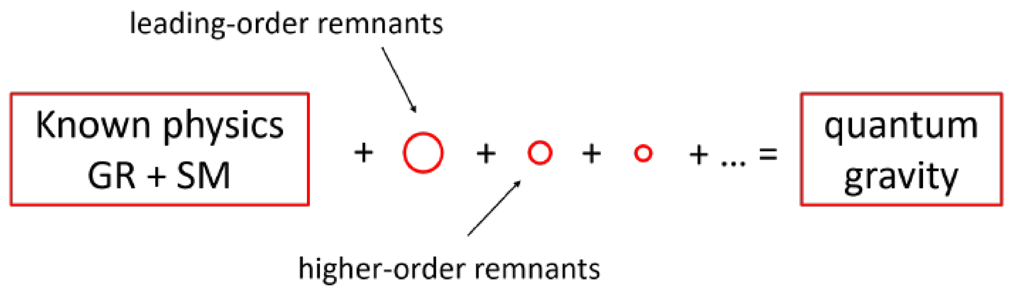

Figure 1. Depiction of the effective-field theory nature of the SME; credit: Matthew Mewes, Cal Poly

The Laser Interferometer Space Antenna (LISA) mission is the European Space Agency future space based gravitational-wave detector, which will be highly sensitive to low-frequency gravitational waves in the band $< 10^{-4}$ Hz to $> 10^{-1}$ Hz. Within this band lie a multitude of Galactic sources comprised of white dwarfs and neutron stars in different combinations, known as **Galactic Binaries**. These non-coalescing, relatively slow-moving sources emit continuous, quasi-monochromatic gravitational waves with a period of minutes to hours which will be **observable by LISA throughout the entire mission lifetime**. The fact that these are "weak" and slow-moving sources means that they can be treated using a Post-Newtonian expansion, without the need to employ numerical relativity and computationally expensive waveform modelling. These sources are of significantly lower energy than the mergers detected by ground-based detectors, but they are plentiful and continuously observable, and so the amount of statistics which LISA can gather will be considerable.

Effective-Field Theory Setup

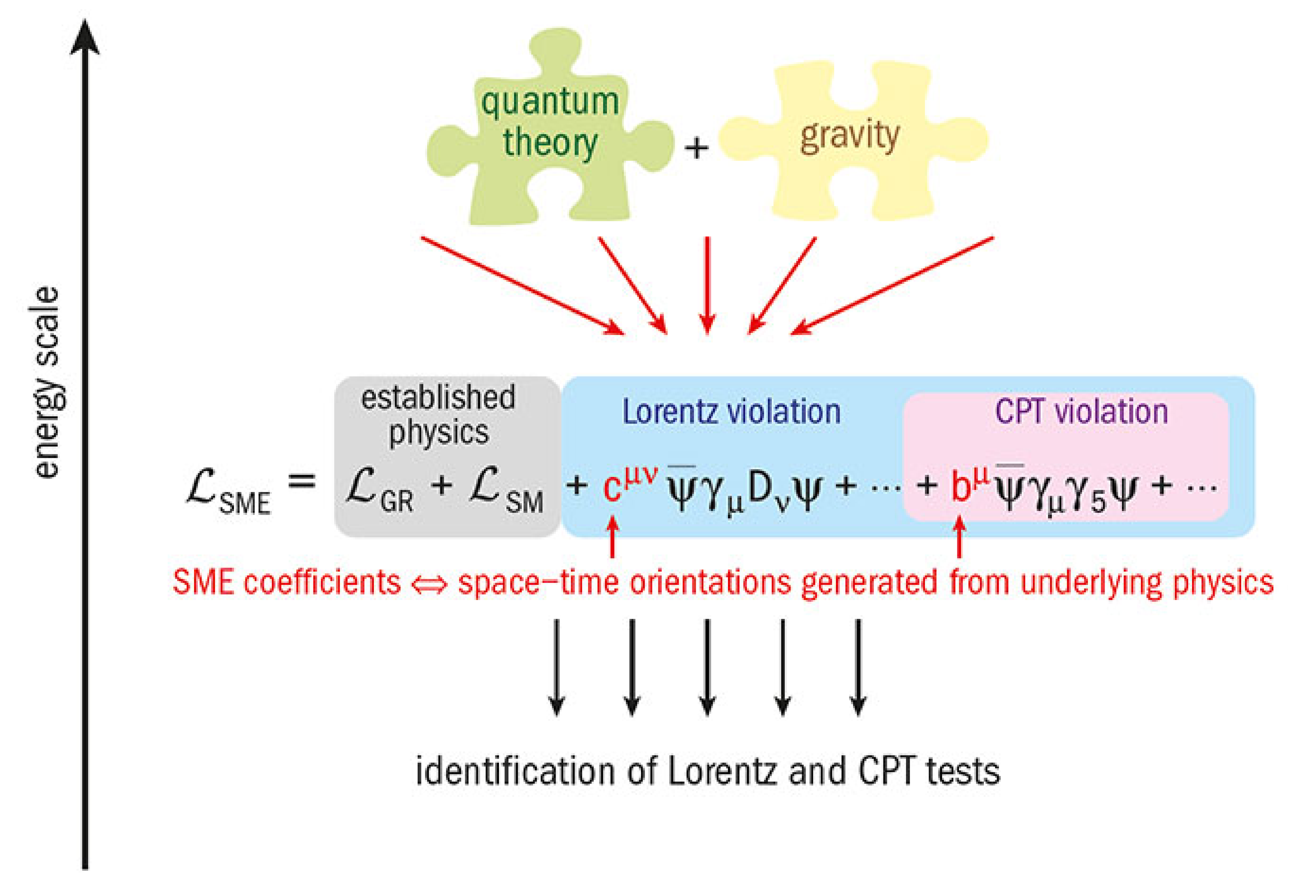

Figure 2. Schematic picture of the effective-field theory; credit: Ralf Lehnert (IUCSS)

Lagrange density

$$\mathcal{L} = \frac{1}{8\kappa}\epsilon^{\mu\rho\alpha\kappa}\epsilon^{\nu\sigma\beta\lambda}\eta_{\kappa\lambda}h_{\mu\nu}\partial_\alpha\partial_\beta h_{\rho\sigma} + \frac{1}{8\kappa}h_{\mu\nu}\left(\hat{s}^{\mu\rho\nu\sigma} + \hat{q}^{\mu\rho\nu\sigma} + \hat{k}^{\mu\rho\nu\sigma}\right)h_{\rho\sigma}$$

- General Relativity
- Symmetry-breaking contribution

Field equations

$$G_L^{\mu\nu} + M^{\mu\nu\rho\sigma}h_{\rho\sigma} - \frac{\kappa}{c^4}\tau^{\mu\nu} = 0$$

Solution scheme [3, 4]

- Adopt an order-by-order solution scheme, where GR is the zeroth order

$$\bar{h}^{\mu\nu} = \bar{h}^{(0)\mu\nu} + \bar{h}^{(1)\mu\nu}$$

$$\Box\bar{h}^{(0)\mu\nu} = -\frac{2\kappa}{c^4}\tau^{\mu\nu} \qquad \Box\bar{h}^{(1)\mu\nu} = 2\bar{M}^{\mu\nu\rho\sigma}\bar{h}^{(0)}_{\rho\sigma}$$

$$\bar{h}^{(0)\mu\nu}(x) = \frac{\kappa}{4\pi c^4}\int d^4y\, G(x-y)\tau^{\mu\nu}(y)$$

$$\boxed{\bar{h}^{(1)\mu\nu} = -\frac{\kappa}{8\pi^2 c^4}\int d^4y d^4z G(x-y)G(y-z)\bar{M}^{\mu\nu\alpha\beta}\tau_{\alpha\beta}(z)}$$

Integration regions and solution algorithm

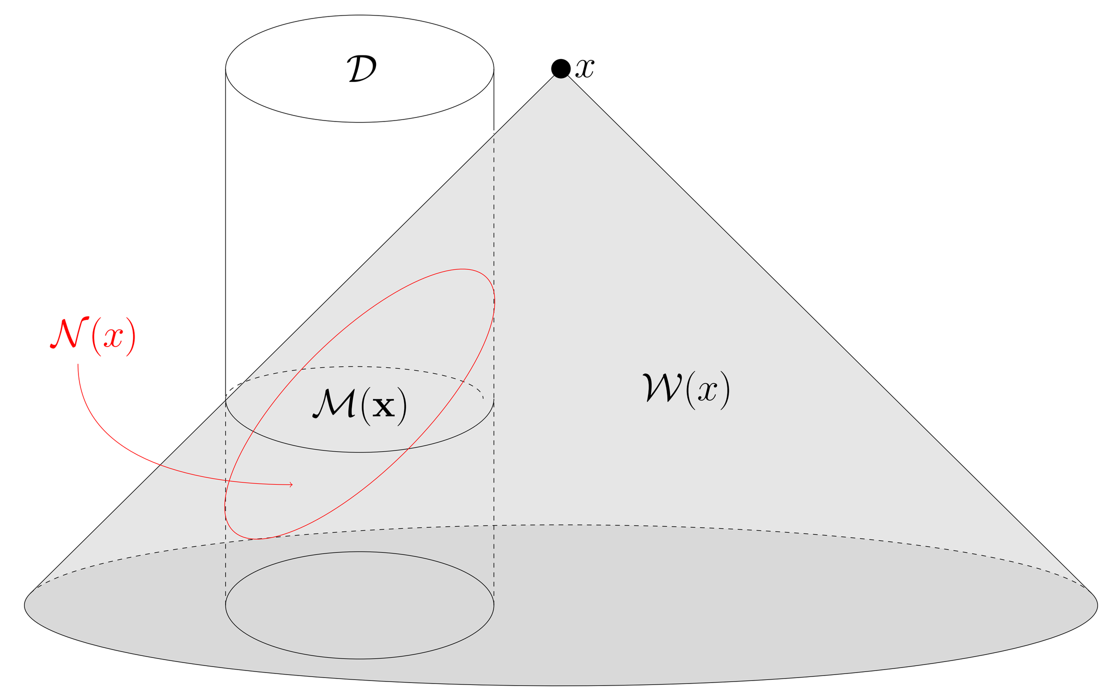

Figure 3. The past lightcone $\mathcal{C}(x)$ of the field point x, where $\mathcal{D}$ is the world tube traced by a codimension-1 sphere of radius $\mathcal{R}$. $\mathcal{C}(x)$ is split into the near zone $\mathcal{N}(x)$ (which lies on the surface of the lightcone and is contained within $\mathcal{D}$) and the wave zone $\mathcal{W}(x)$. The constant-time surface $\mathcal{M}(x)$ is the relevant integration region in the near zone..

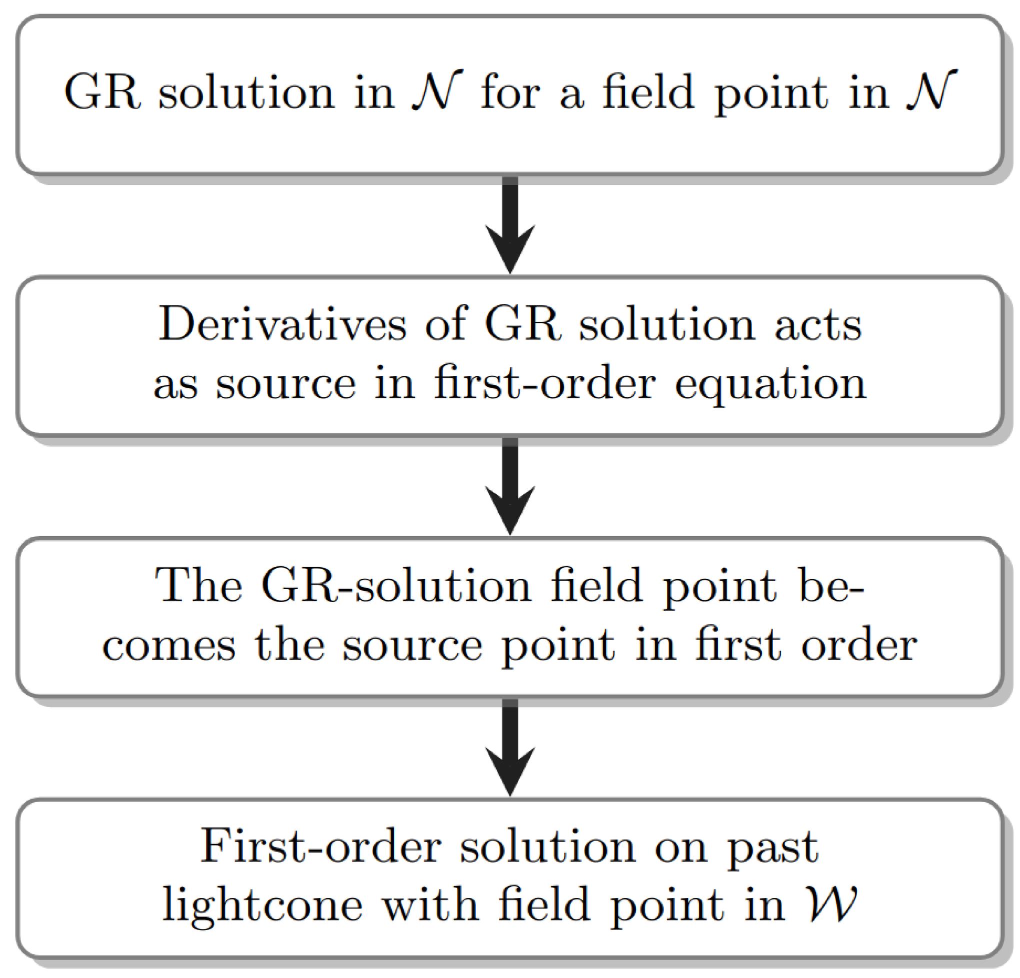

Figure 4. The solution-generating algorithm used. Similar logic applies to the wave-zone solutions, but there we will have an extra contribution from the near zone.

- GR solution in the near-zone takes the form of a Post-Newtonian series as

$$\bar{h}^{(0)00} = \frac{4}{c^2}U + \frac{1}{c^4}\left(7U^2 + 4\psi - 4V + 2\frac{\partial^2 X}{\partial t^2}\right) + \mathcal{O}(c^{-5})$$

- There is a need to count the number of time derivatives in the near zone

$$\bar{M}^{\mu\nu\rho\sigma}\bar{h}^{(0)}_{\rho\sigma} = \partial\,\partial\,\bar{h}^{(0)} + \partial\,\partial\,\partial\,\bar{h}^{(0)} + \ldots$$

- In the near zone, we do a multipole expansion

$$\bar{h}^{(1)\mu\nu}(x) = -\frac{1}{2\pi r}\sum_{\ell=0}^{\infty}\frac{n_L}{\ell! c^\ell}\left(\frac{d}{d\tau}\right)^\ell \int_{\mathcal{M}} d^3x' \bar{M}^{\mu\nu\alpha\beta}h^{(0)}_{\alpha\beta}(\tau, \mathbf{x}')x'^L$$

Toy Solution

- Point particles and a simple symmetry-breaking coefficient
- Need to regularise the integrals and apply distributional derivatives

$$\boxed{\bar{h}^{(1)jk} \supseteq \bar{h}^{\mathrm{GR}jk}_{\mathcal{N}\mathcal{W}} - \frac{4G}{3c^4 r}\tilde{\bar{s}}^{jkmi}\ddot{I}^{\mathrm{GR}}_{im} + \mathcal{O}(c^{-5})}$$

- Solution proportional to known GR objects!

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