Precise positioning using carrier-phase measurements in a swarm of satellites

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1. Context

A distributed instrument in space missions consists of multiple satellite nodes that must operate as a single entity by merging their measurements.

This demands accurate positioning of each satellite, both globally and within the group, which typically involves measuring the time delay (τ) between nodes. Finer time-delay measurements lead to more accurate positioning, which can be enhanced by considering the impact of τ on the signal's carrier phase.

This thesis aims to characterize the impact in estimating τ under such conditions through performance assessments.

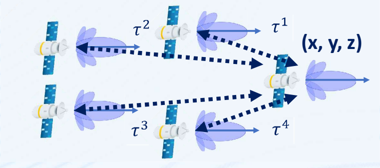


Figure 1 °

2. Signal model

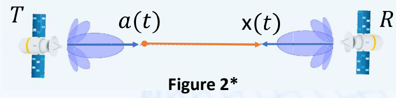


Figure 2*

$$x(t) = \left(\alpha e^{j\phi}\right) a(t;\boldsymbol{\eta}) e^{-jw_c b(t-\tau)} + n(t),$$
$$\phi = \psi - w_c\tau, \quad \boldsymbol{\eta} = (b,\tau), \quad n(t) \sim CN(0,\sigma_n^2), \quad \alpha \in \mathbb{R}$$

- **Evidence:**
  Phase ψ and $w_c\tau$ are mixed up together as a single entity.
- **Hypothesis:**
  If ψ is compensated, $w_c\tau$ can be included in the signal model allowing for:
  - more representative signal model
  - more accurate estimation performance assessment of τ.
- **Methods:** Devise strategies dealing with ψ term,

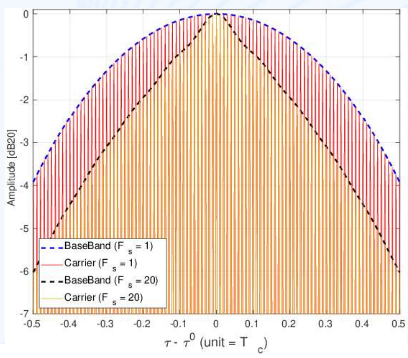
3. Methods

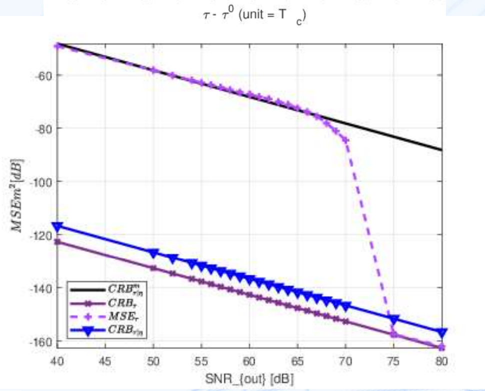
Conduct performance assessment by deriving the CRLB [a] and MLE [b], based on assumptions on the signal model.

A. Static model [1]

Doppler b and ψ known and compensated

$$x(t) = \alpha a(t-\tau) e^{-jw_c\tau} + n(t)$$

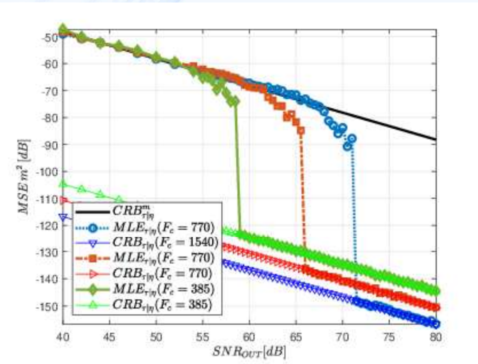


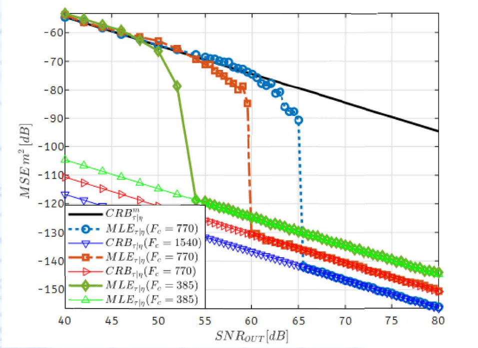


B. Dynamic model [2]

b unknown and ψ compensated

$$x(t) = \alpha a(t;\boldsymbol{\eta}) e^{j\varphi(\boldsymbol{\eta})} + n(t)$$
$$\varphi(\boldsymbol{\eta}) = w_c\,(\tau + b(t-\tau))$$

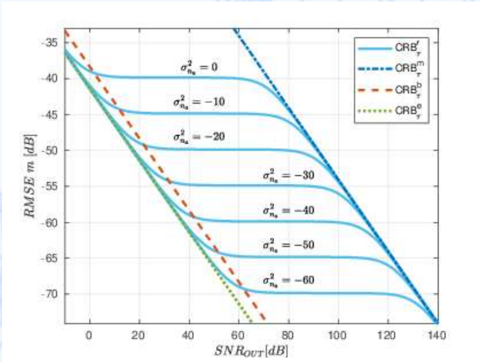


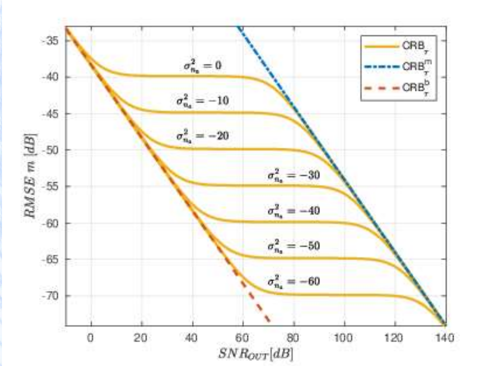


C. Dynamic model + comp.

b unknown and ψ estimated & compensated

$$x(t) = \alpha a(t;\boldsymbol{\eta}) e^{j\varphi(\boldsymbol{\eta})} e^{(\psi - \hat{\psi})} + n(t)$$
$$\hat{\psi} = \psi + n_a(t), \quad n_a(t) \sim N(0,\sigma_a^2)$$





a. CRLB: Cramér-Rao Lower Bound

b. MLE: Maximum-Likelihood Estimator

°. **Figure 1:** Satellite computes its coordinates from differences in signal's time of arrival τ^k

*. **Figure 2:** Transmitter (T) sends $a(t)$ and receiver (R) receives $x(t)$

References

- [1] J. M. Bernabeu Frias, L. Ortega, A. Blais, Y. Gregoire and E. Chaumette, "Time-Delay and Doppler Estimation with a Carrier Modulated by a Band-Limited Signal," *2023 IEEE 9th International Workshop on Computational Advances in Multi-Sensor Adaptive Processing (CAMSAP)*, doi: 10.1109/CAMSAP58249.2023.10403430.
- [2] Bernabeu, J.M., Ortega, L., Blais, A. et al. On the asymptotic performance of time-delay and Doppler estimation with a carrier modulated by a band-limited signal. EURASIP J. Adv. Signal Process. 2024, 47 (2024). https://doi.org/10.1186/s13634-024-01134-2