Multispectral and Hyperspectral Image Fusion with JWST/MIRI

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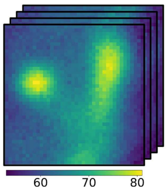
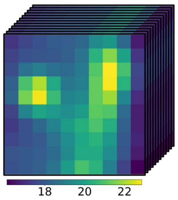
Context

- **Data** : multispectral cube acquired by an imager, hyperspectral cube by a spectrometer



Multispectral data* $\boldsymbol{y}_\mathrm{m}$

  - **High** spatial resolution
  - **Low** spectral resolution



Hyperspectral data* $\boldsymbol{y}_\mathrm{h}$

  - **Low** spatial resolution
  - **High** spectral resolution

- **Objective** : restoration of a cube $\boldsymbol{x}$ with **enhanced** spatial and spectral resolutions.

- **Hypotheses** :
  - Subspace approximation with the Linear Mixing Model [3] [4], such that

$$\boldsymbol{x}[i,j,l] = \sum_{t=1}^{T} \boldsymbol{a}_t[i,j]\ \boldsymbol{s}_t[l]$$

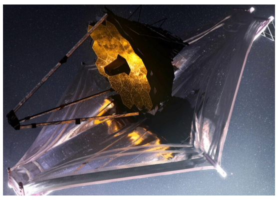
$$\boldsymbol{x} = \boldsymbol{T}\boldsymbol{a}$$

  - Data corrupted with additive white gaussian noises.
  - The imager and spectrometer models are known [3] [4].

*simulated observations of the Orion Bar [1] [2]

Instrument models

- Application to the Mid-InfraRed Instrument (MIRI) of the James Webb Space Telescope (JWST)
- MIRI has 2 components :
  - an imager, MIRIM,
  - an integral field spectrometer, the MRS.
- Spectral range of MIRI : 5 to 28 µm.



JWST

- **Imager model**
  - Spectrally varying spatial blur, i.e. convolution with the imager impulse response $\boldsymbol{h}_\mathrm{m}$
  - Spectral response of the imager $\boldsymbol{w}_\mathrm{m}$
  - Spectral integration over C bands
  - Forward model $\Longrightarrow$ $\boldsymbol{y}^c_\mathrm{m}[i,j] = \sum_{l} (\boldsymbol{x} \underset{i,j}{*} \boldsymbol{h}_\mathrm{m})[i,j,l]\ \boldsymbol{w}^c_\mathrm{m}[l] + \boldsymbol{n}^c_\mathrm{m}[i,j]$

$$\boldsymbol{y}_\mathrm{m} = \underbrace{\boldsymbol{W}_\mathrm{m}\boldsymbol{C}_\mathrm{m}\boldsymbol{T}}\boldsymbol{a} + \boldsymbol{n}_\mathrm{m}$$

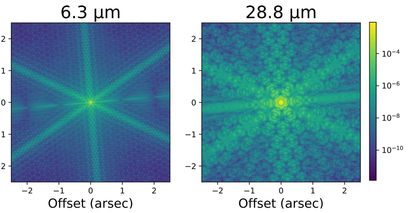
$= \boldsymbol{M}$, forward imager model matrix

- **Spectrometer model**
  - Spectrally varying spatial blur, i.e. convolution with the imager impulse response $\boldsymbol{h}_\mathrm{h}$
  - Spectral response of the spectrometer $\boldsymbol{w}_\mathrm{h}$
  - Spatial subsampling (aliasing)
  - Forward model $\Longrightarrow$ $\boldsymbol{y}_\mathrm{h}[\bar{i},\bar{j},l] = \sum_{i=\bar{i}d_i}^{(\bar{i}+1)d_i} \sum_{j=\bar{j}d_j}^{(\bar{j}+1)d_j} (\boldsymbol{x} \underset{i,j}{*} \boldsymbol{h}_\mathrm{h})[i,j,l]\ \boldsymbol{w}_\mathrm{h}[l] + \boldsymbol{n}_\mathrm{h}[\bar{i},\bar{j},l]$

$$\boldsymbol{y}_\mathrm{h} = \underbrace{\boldsymbol{S}\overline{\boldsymbol{C}_I}\boldsymbol{W}_\mathrm{h}\boldsymbol{C}_\mathrm{h}\boldsymbol{T}}\boldsymbol{a} + \boldsymbol{n}_\mathrm{h}$$

$= \boldsymbol{H}$, forward spectrometer model matrix

Impulse response of JWST/MIRI



Methodology

- Case of an **ill-posed inverse problem**, solved by minimizing a regularized convex criterion

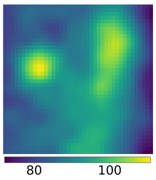
$$\hat{\boldsymbol{a}} = \underset{\boldsymbol{a}}{\mathrm{argmin}} \left\{ \underbrace{\mu_\mathrm{m}\|\boldsymbol{y}_\mathrm{m} - \boldsymbol{M}\boldsymbol{a}\|_2^2 + \mu_\mathrm{h}\|\boldsymbol{y}_\mathrm{h} - \boldsymbol{H}\boldsymbol{a}\|_2^2}_{\text{Data adequation}} + \underbrace{\mu_\mathrm{r}R(\boldsymbol{a})}_{\text{Regularization}} \right\}$$

- Two regularizations used : quadratic (ℓ_2-norm) $R(\boldsymbol{a}) = ||\boldsymbol{D}\boldsymbol{a}||^2$; half-quadratic ($\ell_{2,1}$-norm) [6] $R(\boldsymbol{a}) = \varphi(\boldsymbol{D}\boldsymbol{a})$
- Both cases : resolution of a linear system $\boldsymbol{Q}\hat{\boldsymbol{a}} = \boldsymbol{q}$, solved in the literature [5] with gradient based algorithms for the ℓ_2 -norm, where

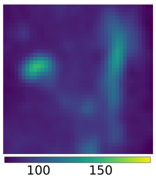
$$\boldsymbol{Q} = \mu_\mathrm{m}\boldsymbol{M}^H\boldsymbol{M} + \mu_\mathrm{h}\boldsymbol{H}^H\boldsymbol{H} + \mu_\mathrm{r}\boldsymbol{D}^H\boldsymbol{D}$$

Contribution

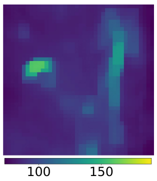
- Proposed procedure for the **fast** and **exact** calculation of $\boldsymbol{Q}^{-1}$ by demonstrating its diagonal block structure using [7] and applying a matrix inversion method from [3].
- Two main contributions :
  - the fast calculation of the **exact solution** for ℓ_2, with $\hat{\boldsymbol{a}} = \boldsymbol{Q}^{-1}\boldsymbol{q}$,
  - an **accelerated** procedure for the alternating minimization problem [3][4] for $\ell_{2,1}$.



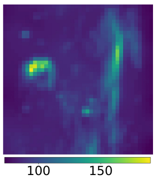
Coaddition



Exact solution of ℓ_2 [5]



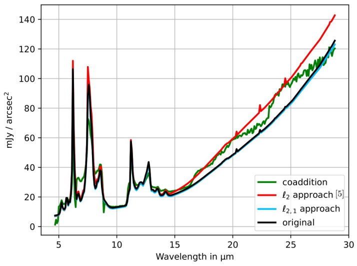
Proposed $\ell_{2,1}$ approach



Original

Methods	NRMSE ($\times 10^{-3}$)	dSSIM ($\times 10^{-5}$)	SAM ($\times 10^{-3}$)	PSNR	Time [s]
Coaddition	133	1476	119	37	**0.6**
Exact solution of ℓ_2 [5]	27	241	5.8	50	2 (with prep.)
Proposed $\ell_{2,1}$ approach	**22**	**179**	**4.0**	**52**	19 (300 iter.)

- Efficient **deconvolution** and **denoising** for all wavelength with inverse problem approaches, mainly thanks to correlations induced by the Linear Mixing Model
- Exact solution of ℓ_2 **1000 times faster*** than minimization with gradient based algorithm [5] for a low noise case (SNR = 100 dB)
- Best spatial and spectral resolutions found with the proposed edge-preserving $\ell_{2,1}$ approach



*Size MS dataset : 9 x 124 x 248, Size of HS dataset :300 x 31 x 62, Size of reconstruction : 300 x 124 x 248.

Acknowledgments

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References

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