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Dissipation mechanisms of the inner core's translational oscillations

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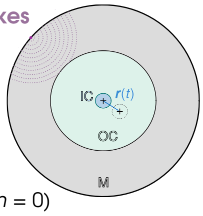
Probing of the Earth's interior is limited to a few measurements. For example, a more accurate estimate of the density difference between the inner and outer core could better constrain the driving force of the geodynamo. Gravimetric measurements of the **translational oscillations of the inner core** could help in this respect, but these oscillations still elude detection. **Translational oscillations**, also known as **Slichter modes**[1,2], are the result of extreme events, such as massive earthquakes or asteroid impacts, which can slightly **displace the inner core**. The centre of mass of the inner core would later swing around the equilibrium position as a **damped oscillator**.

Previous linear models could only predict the **oscillation period**[3,4], bounding the frequency range of interest for observations. Here, for the first time, we study the **viscous and magnetic dissipation mechanisms** through non-linear simulations of the **outer core fluid response**. We take full advantage of the spherical shell geometry and use the fast pseudo-spectral code XSHELLS[5] to solve the problem numerically. Since the study of realistic Earth values is out of reach, we use a systematic exploration of the parameter space to derive **scaling laws** that can be used to extrapolate to Earth conditions.

Translational oscillations[1,2]

- **Strong Earthquakes**
- **Impact events**

can displace the **inner core**.



The **inner core** center of mass evolution $\boldsymbol{r}(t)$ follows Newton's 2nd law[3,4]:

$$m_{ic}\frac{\mathrm{d}^2\boldsymbol{r}}{\mathrm{d}t^2} = \sum_j \boldsymbol{F}_j$$

- Coriolis - centrifugal
- gravity - added mass
- viscous - magnetic

Modes splitting:
- 1 **polar** ($l = 1, m = 0$)
- 2 **equatorial** ($l = 1, m = -1,+1$)

Known **period** T_{OSC} = 4-6 h, but unclear **dissipation** mechanisms.

Physical model implemented in XSHELLS[5]

Incompressible Navier-Stokes in the **outer core**

$$\frac{\partial \boldsymbol{u}}{\partial t} + \boldsymbol{u}\cdot\nabla\boldsymbol{u} + 2\mathbf{e}_z\times\mathbf{u} = -\nabla p^* + Ek\nabla^2\boldsymbol{u} + Le^2(\nabla\times\mathbf{b})\times\mathbf{b}\ , \quad \nabla\cdot\boldsymbol{u} = 0$$

Induction equation in the **inner** and the **outer core**

$$\frac{\partial \mathbf{b}}{\partial t} = \nabla\times(\boldsymbol{u}\times\mathbf{b}) + \underbrace{EkPm^{-1}}_{E\eta}\nabla^2\mathbf{b}\ , \quad \nabla\cdot\mathbf{b} = 0$$

Velocity b.c. at the **inner core boundary** $u_r(R_i) = \varepsilon(t)Y_1^m(\theta,\varphi)$

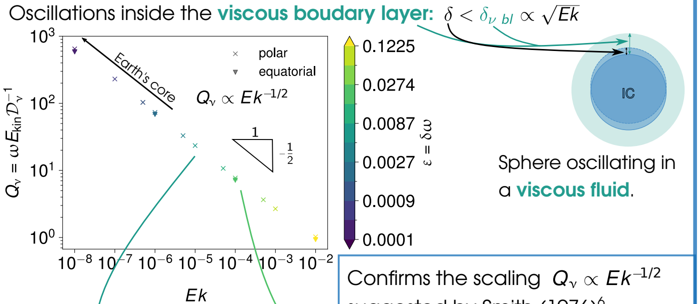
INPUTS:		Earth core
dimensionless displacement:	$\delta = \frac{\breve{\delta}_{osc}}{R_o}$	10^{-8}
dimensionless frequency:	$\omega = \frac{\breve{\omega}_{osc}}{\Omega_s}$	6
Ekman number:	$Ek = \frac{\nu}{\Omega_s R_o^2}$	10^{-15}
magnetic Prandtl number:	$Pm = \frac{\nu}{\eta_m}$	10^{-6}
Lehnert number:	$Le = \frac{B_0}{\sqrt{\rho\mu_0}\Omega_s R_o}$	10^{-4}
aspect ratio:	$\Gamma = \frac{R_i}{R_o}$	0.35

Forced oscillations: $\varepsilon(t) = \varepsilon\cos(\omega t)$

OUTPUTS:
- kinetic energy: $E_k = \frac{1}{2}\int |\boldsymbol{u}|^2\,\mathrm{d}V$
- viscous dissipation: $\mathcal{D}_\nu = \nu\int_\Omega |\nabla\times\boldsymbol{u}|^2\mathrm{d}V$
- magnetic dissipation: $\mathcal{D}_\eta = \eta_m\int_\Omega |\nabla\times\mathbf{b}|^2\mathrm{d}V$
- quality factor: $Q_i = E_{kin}\omega\mathcal{D}_i^{-1}$

Viscous dissipation

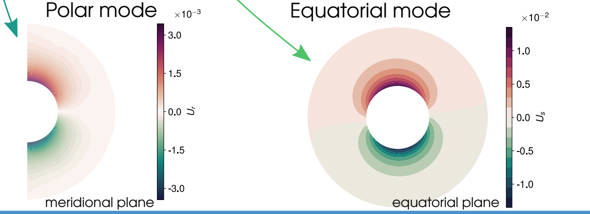
Oscillations inside the **viscous boudary layer**: $\delta < \delta_{\nu\ bl} \propto \sqrt{Ek}$



Sphere oscillating in a **viscous fluid**.

Confirms the scaling $Q_\nu \propto Ek^{-1/2}$ suggested by Smith (1976)[6]

our estimate **for the Earth**: $Q_\nu \approx 10^6$

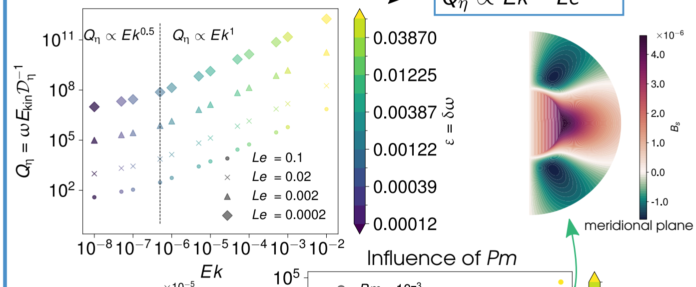


Magnetic dissipation of the polar mode

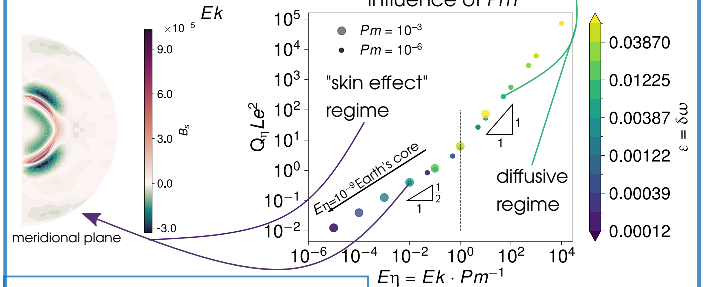
Forced **polar** oscillation: $u_z = \varepsilon\cos(\omega t)$

Uniform vertical magnetic field: $\mathbf{b} = B_0\cdot\mathbf{e}_z$

Influence of Ek and Le → $Q_\eta \propto Ek^{1/2}Le^{-2}$



Influence of Pm



$$Q_\eta \propto Ek^{1/2}Le^{-2}Pm^{-1/2}$$

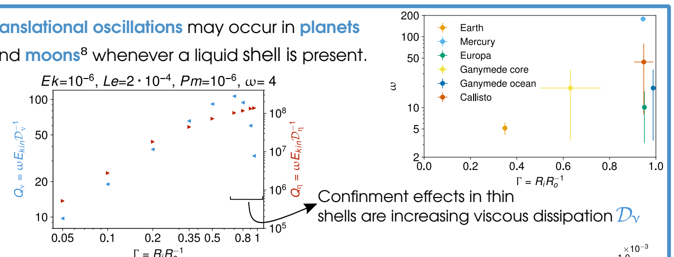
our estimate **for the Earth**: $Q_\eta \approx 7\cdot 10^3$

Buffet et al. (1995)[7]: $2\cdot 10^3 < Q_\eta < 5.8\cdot 10^5$

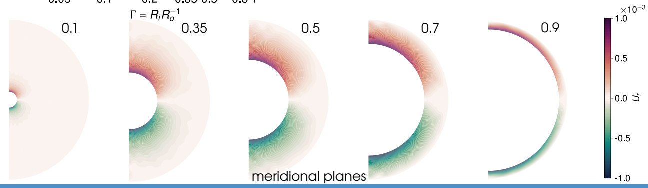
Magnetic dissipation is the largest: $\mathcal{D}_\eta \gg \mathcal{D}_\nu$

Influence of the inner radius

Translational oscillations may occur in **planets** and **moons**[8] whenever a liquid shell is present.



Confinment effects in thin shells are increasing viscous dissipation $\mathcal{D}_\nu$



References:

[1] L.B. Slichter, "The fundamental free mode of the Earth's inner core", PNAS 1961
[2] S. Rosat, "A review of the Slichter modes: an observational challenge", 2012
[3] F.H. Busse, "On the free oscillation of the Earth's inner core" JGR, 1974
[4] M. Rieutord, "Slichter modes of the Earth revisited." PEPI, 2002
[5] N. Schaeffer, "Efficient spherical harmonic transforms aimed at pseudospectral numerical simulations," G3, 2013
[6] M.L. Smith, "Translational inner core oscillations of a rotating, slightly elliptical Earth." JGR, 1976
[7] B.A. Buffett, and D.E. Goertz, "Magnetic damping of the translational oscillations of the inner core." GJI, 1995
[8] A. Coyette, and T. Van Hoolst, "Slichter modes of large icy satellites." Icarus, 2014

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