Detectability of higher harmonics with LISA

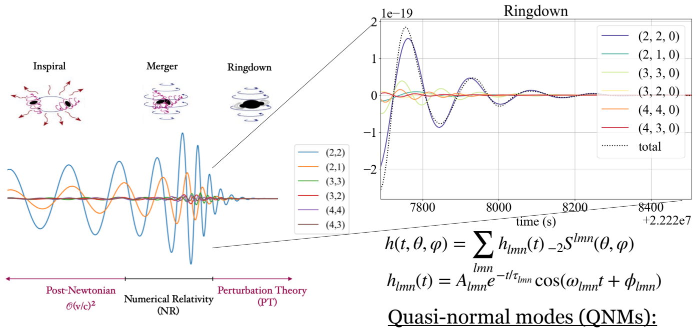
Chantal Pitte

Quentin Baghi, Marc Besancon, Sylvain Marsat and Antoine Petiteau

based on DOI: 10.1103/PhysRevD.108.044053

Motivation

- Quantify our ability to identify higher modes in the emission of gravitational waves (GW) of supermassive black hole binaries (SMBHB).
- Large signal-to-noise ratio (SNR) in LISA enhances the possibility to test General Relativity (GR) ⟶ No-hair theorem.
  - BHs are characterised by the mass and the spin BH(M_f, a_f).



$$h(t,\theta,\varphi) = \sum_{lmn} h_{lmn}(t)\, {}_{-2}S^{lmn}(\theta,\varphi)$$

$$h_{lmn}(t) = A_{lmn} e^{-t/\tau_{lmn}} \cos(\omega_{lmn} t + \phi_{lmn})$$

Quasi-normal modes (QNMs):

$$\tilde{\omega}_{lmn} = \omega_{lmn}(M_f, a_f) + i/\tau_{lmn}(M_f, a_f)$$

Is crucial to correctly estimate the SMBHB parameters!!!

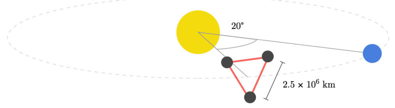
- Inspiral-merger-ringdown (IMR) waveform can be decomposed in spherical harmonics:

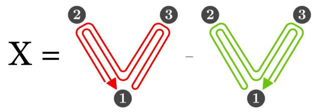
$$h_+ - ih_\times = \sum_{l\geq 2} \sum_{m=-l}^{l} {}_{-2}Y_{lm}(\iota,\varphi)\, h_{lm} \qquad h_{lm}^{IMR}(f) = A_{lm}(f)\, e^{-i\phi_{lm}(f)}$$

LISA response

LISA: Laser Interferometer Space Antenna

- 3 space-crafts in a triangular constellation in Earth's orbit.
- Time delay interferometry (TDI), to cancel laser noise:





$$\begin{cases} A = \frac{1}{\sqrt{2}}(Z - X) \\ E = \frac{1}{\sqrt{6}}(X - 2Y + Z) \\ T = \frac{1}{\sqrt{3}}(X + Y + Z) \end{cases}$$

- LISA response can be integrated as a transfer function in Fourier's domain:

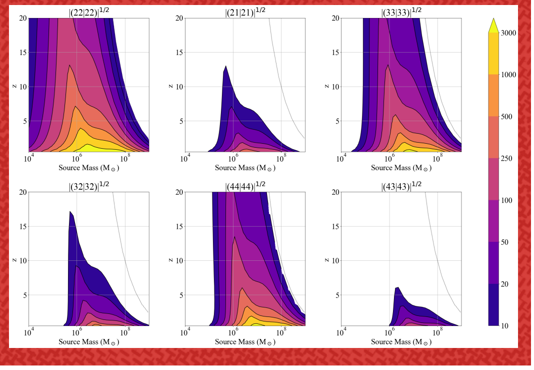
$$\mathcal{H}^I_{lm}(f) = \mathcal{T}^I_{lm}(f) h_{lm}(f)$$

Modes contribution to the SNR and mass dependency

The detectability of the modes is related to their SNR. Each mode SNR depends on the frequency and thus on the mass of the event.

$$\rho^2 = \sum_{lm}\sum_{l'm'} (lm\,|\,l'm')$$

$$(lm\,|\,l'm') = \sum_{I=a,e} 4\Re \int \frac{\mathcal{H}^I_{lm}(f)\mathcal{H}^{I\,*}_{l'm'}(f)}{S_n(f)} df$$



Results with different models for M_6 injection

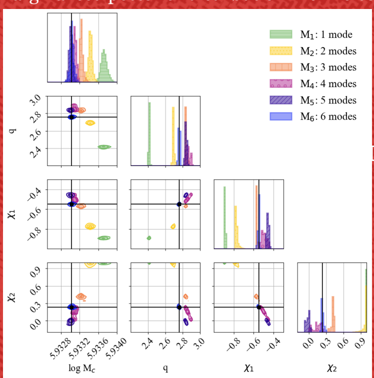
Models definition

Model	Modes (l,m)
M_1	(2, 2)
M_2	(2, 2), (3, 3)
M_3	(2, 2), (3, 3), (4, 4)
M_4	(2, 2), (3, 3), (4, 4), (2, 1)
M_5	(2, 2), (3, 3), (4, 4), (2, 1), (3, 2)
M_6	(2, 2), (3, 3), (4, 4), (2, 1), (3, 2), (4, 3)

$$\mathcal{B} = \frac{\mathcal{Z}_i}{\mathcal{Z}_j}, \text{ where } \mathcal{Z} = \int_\Theta \mathcal{L}(\theta)\pi(\theta)d\theta$$

Bayes factor	Noiseless data set	Noisy data set
$\log(\mathcal{Z}_1/\mathcal{Z}_6)$	-6845	-6873
$\log(\mathcal{Z}_2/\mathcal{Z}_6)$	-976	-1015
$\log(\mathcal{Z}_3/\mathcal{Z}_6)$	-237	-259
$\log(\mathcal{Z}_4/\mathcal{Z}_6)$	-109	-134
$\log(\mathcal{Z}_5/\mathcal{Z}_6)$	-84	-100

Marginalised posterior distribution with noise



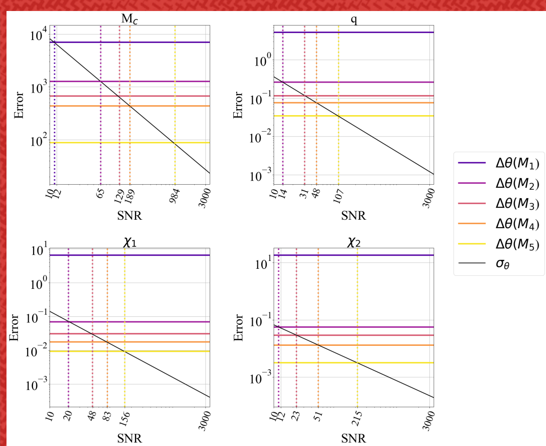
Estimated values for models M_1 and M_6

Parameter	True value	Estimated value with M_1 (noiseless)	Estimated value with M_6 (noiseless)	Estimated value with M_1 (with noise)	Estimated value with M_6 (with noise)
$\log M_c$ ($M_\odot$)	5.93302	$5.93371^{+0.00019}_{-0.00016}$	$5.93303^{+0.00010}_{-0.00010}$	$5.93374^{+0.00019}_{-0.00016}$	$5.93304^{+0.00009}_{-0.00010}$
q	2.759	$2.411^{+0.012}_{-0.011}$	$2.751^{+0.019}_{-0.021}$	$2.414^{+0.012}_{-0.012}$	$2.759^{+0.013}_{-0.023}$
χ_1	-0.549	$-0.890^{+0.009}_{-0007.}$	$-0.559^{+0.016}_{-0.024}$	$-0.888^{+0.009}_{-0.008}$	$-0.549^{+0.011}_{-0.021}$
χ_2	0.232	$0.996^{+0.004}_{-0.019}$	$0.261^{+0.064}_{-0.043}$	$0.996^{+0.004}_{-0.018}$	$0.231^{+0.057}_{-0.030}$

Modelling error and SNR dependency

Using an incorrect template results in a systematic modelling error ($\Delta\theta_i$). If the statistical error (σ_{θ_i}) is smaller than the modelling error, the bias in the parameters becomes relevant.

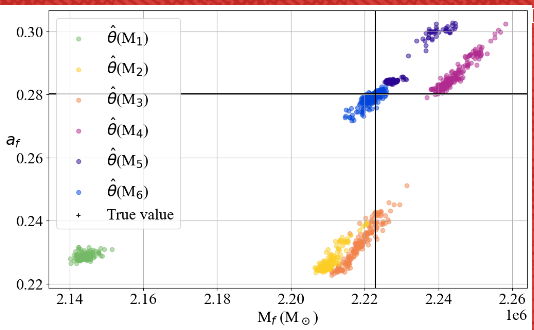
$$\begin{cases} \sigma_{\theta_i} = \sqrt{\Gamma^{-1}_{ii}} \\ \Delta\theta_i = \sum_j \Gamma^{-1}_{ij}\left(\frac{\partial h}{\partial \theta_j}\,\middle|\,\delta h\right) \end{cases}$$



Impact of biased parameters

One can obtain the values of the final mass and final spin of the remnant BH from the parameters of the progenitors.

The bias in the estimated parameters will translate into bias in the final BH's parameters.



We need the accurate value of final mass and final spin in order to test the 'no-hair' theorem.

Conclusions

- Given the redshift and the source mass of an event we can infer the relevance of the modes through the map-guide of SNR presented here.
- We are able to discriminate models and therefore modes with a Bayesian analysis.
- We see how the use of an incorrect template of modes causes bias in the parameter estimation.
- Given a certain SNR we can constrain the number of modes needed to estimate the parameters without significant bias, in the case of a waveform with 6 modes.
- Biased parameters can lead to misinterpretation in GR tests.