Inference of quasi-geostrophic motions from the geomagnetic secular variation

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INTRODUCTION

Satellite data have revealed oscillations in the magnetic field originating from the core on interannual time-scales (e.g., Chulliat et al, 2014). At these periods, Magneto-Coriolis modes have been numerically described (Gerick et al., 2021), where the magnetic perturbation attaches to a potential field at the core surface. Similar modes have since been detected from magnetic observations (Gillet et al., 2022). We derive the conditions on the core surface flow for the induced radial electrical currents to vanish and magnetic field to satisfy a potential field at the core-mantle boundary. We plan to impose them when inverting for core surface motions from observed magnetic variations.

COMPLEX BASIS

We make use of Generalized Spherical Harmonics(Greff-Lefftz et al, 1995), which allows to replace spatial derivatives by a projection on

$$\mathbf{e}_{\pm} = \frac{1}{\sqrt{2}}(\mp\mathbf{e}_{\theta} - i\mathbf{e}_{\varphi}), \qquad \mathbf{e}_0 = \mathbf{e}_r$$

$$v^{\pm} = \frac{1}{\sqrt{2}}(\mp v^{\theta} - iv^{\varphi}), \qquad v^0 = v^r$$

$$\mathrm{u}^{\pm} = \sum_{l=1}^{L}\sum_{m=-l}^{l} \mathrm{u}_l^{\pm,m}\mathrm{Y}_l^{\pm1,m}$$

The decomposition $\mathbf{v} = v^{+}\mathbf{e}_{+} + v^{-}\mathbf{e}_{-} + v^{0}\mathbf{e}_0$, where v^{+} and v^{-} are developed on a basis of generalized spherical harmonics turns out to be especially useful when v is not solenoidal.

Generalized Spherical Harmonics are defined as

$$\mathrm{Y}_l^{\pm N,m}(\mu,\varphi) = (-1)^{l-N}\frac{\sqrt{2l+1}}{2^l(l-N)!}\sqrt{\frac{(l-N)!\,(l+m)!}{(l+N)!\,(l-m)!}}(1-\mu)^{\left(-\frac{m-N}{2}\right)}$$
$$\times(1+\mu)^{\left(-\frac{m+N}{2}\right)}\frac{d^{l-m}}{d\mu^{l-m}}\left((1-\mu)^{l-N}(1+\mu)^{l+N}\right)e^{im\varphi}$$

where $\mu = \cos(\theta)$. Projection on generalized spherical harmonics functions enables us to avoid the calculation of horizontal (θ,φ) derivatives.

RADIAL INDUCTION EQUATION

Any vector field can be decomposed as: $\mathbf{v} = U(r,\theta,\varphi) + \nabla_1 V(r,\theta,\varphi) + \mathbf{e}_r\times\nabla_1 W(r,\theta,\varphi)$

We can define: $\mathbf{v} = \mathbf{u}\times\mathbf{B}$ $\quad {}^1\mathbf{v} = \nabla\times(\mathbf{u}\times\mathbf{B})$

Using the vector field decomposition and complex basis we can calculate radial induction equation:

$$\forall l \in [1,L], \forall m \in [-l,l]; \left(\frac{\partial B_r}{\partial t}\right)_l^m = {}^1U_l^m = (\nabla\times\mathbf{v})_l^{0,m} = -i\frac{\sqrt{l(l+1)}}{\sqrt{2}r}\left(v_l^{+,m} - v_l^{-,m}\right)$$

$$\mathbf{v}_{\Sigma} = (\mathbf{u}\times\mathbf{B})_{\Sigma} = (u^{+}\mathbf{e}_{+} + u^{-}\mathbf{e}_{-})\times(B_r\mathbf{e}_r) = iB_r(u^{-}\mathbf{e}_{-} + u^{+}\mathbf{e}_{+})$$

And so $v_l^{+,m}$, $v_l^{-,m}$ are calculated as:

$$v^{+} = iB_r u^{+}, \qquad v^{-} = -iB_r u^{-}$$

$$u_l^{\pm,m} = \frac{\sqrt{l(l+1)}}{\sqrt{2}}\left(c_l^m \mp i t_l^m\right)$$

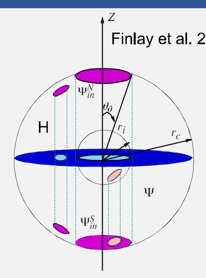
QUASI-GEOSTROPHIC (QG) APPROXIMATION

In rapidly rotating bodies, the dominance of the Coriolis force favors flows aligned with the rotation axis (geostrophy).

Purely geostrophic flows are zonal and possess no variation in the direction parallel to the rotation axis. In spherical geometry, buoyancy and Lorentz forces naturally force non-zonal motions, which present small departures from geostrophy due to mass conservation. The resulting motions are referred to as being quasi-geostrophic.

The QG approximation consists in approaching the actual flow in the core interior by a flow that can be described from a 2D stream function $\psi(\theta,\varphi)$.

The equation for the time evolution of ψ is available. Equatorial symmetry of the estimated core surface flows supports the QG hypothesis.



Finlay et al. 2010

STREAM FUNCTION

The interior QG velocity field is described as a function of the stream function of the stream function in cylindrical coordinates (Schaeffer & Cardin, 2005)

$$\mathbf{u} = \nabla\psi(s,\varphi)\times\left(\frac{z}{H}\right)$$

We follow Holdenried & al (2020), and the stream function ψ can be written in cylindrical and spherical representation that are related to classical spherical harmonic representation. Also, we have the representation between the spherical representation of the stream function and the flow coefficients:

$$\psi = H^3\sum_{m=1}^{L}\sum_{n=0}^{\left[\frac{1}{2}(L-m)\right]} s^m f_{n,m}F_{n,m}(s^2)e^{im\varphi} = H^3\sum_{l=1}^{L}\sum_{m=1}^{l} q_l^m Y_l^m(\theta,\varphi)$$

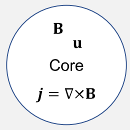
$$2imq_l^m = -l(l+1)c_l^m$$

$$t_l^m = -\frac{1}{l(l+1)}\left(l(l-1)\gamma_{l+1}^m q_{l+1}^m + (l+1)(l+2)\gamma_l^m q_{l-1}^m\right)$$

$$\gamma_l^m = \sqrt{\frac{(l-m)(l+m)}{(2l+1)(2l-1)}}$$

BOUNDARY CONDITIONS ON THE INDUCED FIELD

Two extra constraints on the core surface flow can be derived:



1. No induced toroidal field condition:
Assuming that the mantle is an insulator, poloidal electrical currents must vanish at the core surface, so the toroidal part of induced magnetic field must vanish.

$\boldsymbol{j} = 0 = \nabla\times\boldsymbol{B}$

2. Continuity with a potential field:
The magnetic field has to be continuous at the core surface, so the field from the core side must attach to a potential field.

$j_r = 0$
(is only valid at the Core surface)

We plan to implement them in the core flow inverse problem.

Calculation of the shear:
These constraints depend not only on the flow but also on the shear. And if the flow is QG then the shear can be calculated from the flow.

THE NO TOROIDAL FIELD INDUCTION CONDITION

The no toroidal field induction condition, according to the complex basis and Generalized Spherical Harmonics, can be as the condition on ${}^1W$, so that:

$$\forall l \in [1,L], \forall m \in [0,l], \qquad {}^1v_l^{+,m} = {}^1v_l^{-,m}$$

Or to put it another way:

$$\forall l \in [1,L], \forall m \in [0,l], \qquad \sqrt{2l(l+1)}v_l^{0,m} = \frac{\partial}{\partial r}r\left(v_l^{-,m} + v_l^{+,m}\right)$$

One of the terms include calculation of the radial derivative of horizontal component of the velocity (shear), what can be calculated from the stream function coefficients with the use of QG approximation:

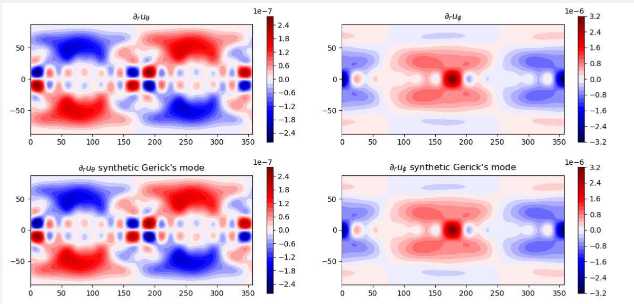
$$\frac{\partial(r\mathbf{u}_{\Sigma})}{\partial r} = \delta^{+}\mathbf{e}_{+} + \delta^{-}\mathbf{e}_{-}$$

Where $\delta^{\pm}$ is calculated as:

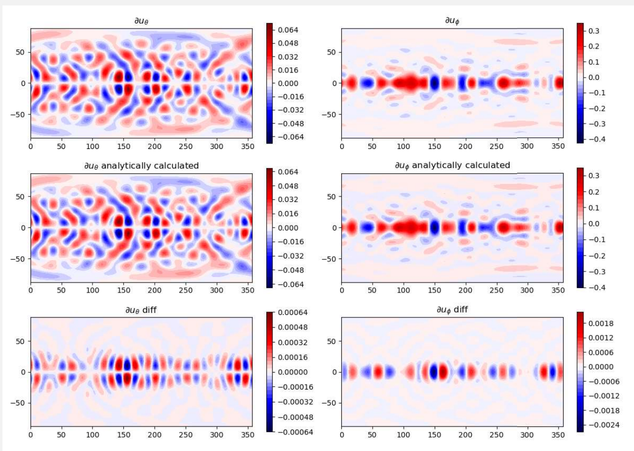
$$\delta^{\pm} = S^{\pm}q_l^m$$

RESULTS

Comparison of maps of shear with the maps of synthetic mode of Felix Gerick:



Comparison of maps shear for Earth Dynamo simulation by Aubert, J., Gillet, N (2021):



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