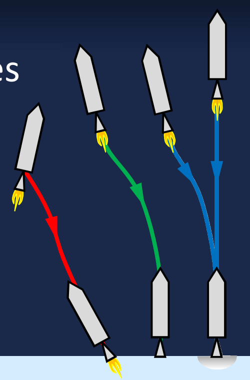
# Online emergency trajectory planning for reusable *tossback* vehicles

Hubert Ménou*, Eric Bourgeois, Nicolas Petit

Thesis prepared at the *Centre Automatique et Systèmes* at Mines Paris
and at the *Direction des Lanceurs* at CNES.



**Keywords:** *Optimal Control, Nonlinear Programming Sensitivity, Path Planning, Safe Set Control, Reusable Rockets*

* hubert.menou@mines-paristech.fr

## A

Emergency landing anywhere near the desired landing site

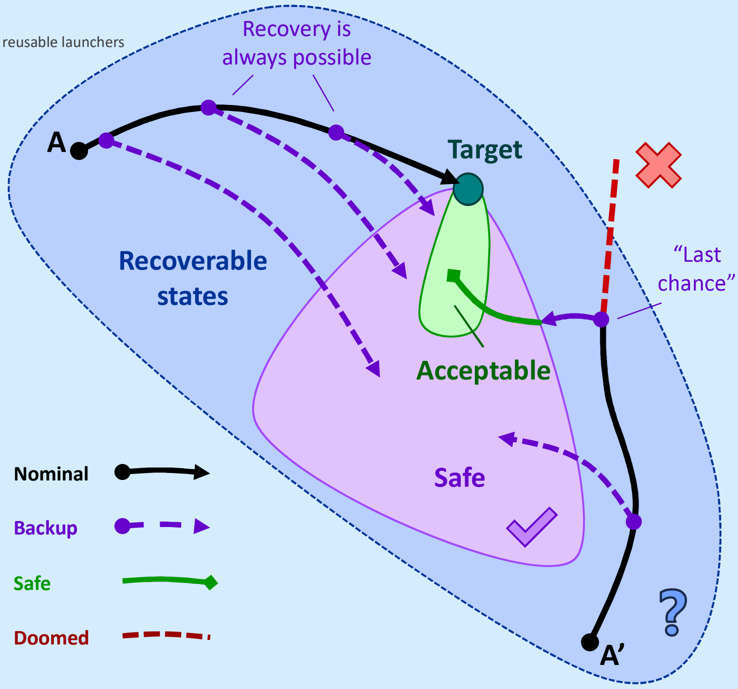
Robustness and theoretical properties of good behavior at run time

For reusable launchers

# How to guarantee null speed rocket landing when something goes wrong?

Initial Position, Speed or Mass, Wind or Pressure profile

Does not have the expected value for the mission




- Rocket landing occurs in a complex and uncertain environment: system states and atmospheric models may often be off nominal when starting the powered landing phase of a flight. Embedded computational power being limited, **fast response to revision in initial conditions and model parameters** is sought for.
- **Typical question:** *How to provide a landing trajectory in-flight when the rocket have 40 kg of fuel less than expected and stronger winds?*
- Generating an optimal trajectory is feasible offline, but very few methods exist nor are fully proven for online use [1].
- It is possible to **supervise** a controller (e.g. any non-proven controller) by simpler proven controllers [2]. The more diverse the simple controllers, the less conservative the approach.
- Our work focuses on **designing those proven controllers**. So far, one generic trajectory-library-based controller and one purely model-based controller have been designed.
These are presented below.

## B

### Trajectory generation via NLP Sensitivity

- Online guidance by 1st-order extrapolation from a library of offline-generated trajectories (implicit final-time).
- **Optimal Control Problem**: small corrections to known references ($x_{ref}$ and $u_{ref}$ prepared offline).

$$\begin{aligned}\min_{\Delta u \in L_\infty, \Delta t_f \in \mathbb{R}} \quad & \frac{1}{2}\|\Delta u\|^2 + \frac{1}{2}\Delta t_f^2 \\ \text{s.t:} \quad & \dot{x}_{ref} + \Delta\dot{x} = f(x_{ref} + \Delta x, u_{ref} + \Delta u, p_0 + \Delta p_1) \\ & \Delta x(0) = \Delta p_2 \\ & u_{ref}(t) + \Delta u(t) \in \mathcal{U}, \quad \forall t \\ & x_{ref}(t) + \Delta x(t) \in \mathcal{X}, \quad \forall t \\ & \Psi((x_{ref} + \Delta x)(t_f + \Delta t_f)) = 0\end{aligned}$$

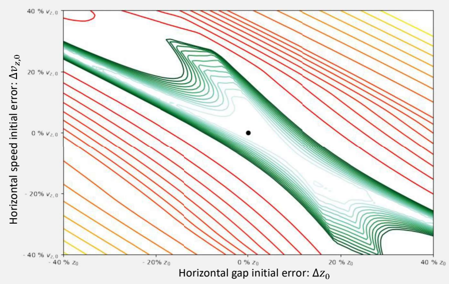
Small parametric changes

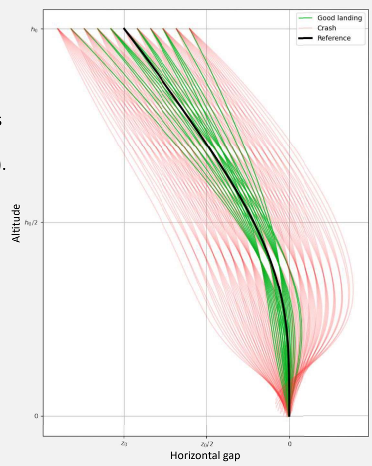
- **Perturbation Methods for Nonlinear Programming**: $\Delta u$ described by a vector variable $z$.

$$\begin{aligned}\min_{z \in \mathbb{R}^{N_z}} \quad & F(z, p_0 + \Delta p) \\ \text{s.t:} \quad & G(z, p_0 + \Delta p) \leq 0\end{aligned} \quad \rightarrow \quad z(p) \approx z_0 + \frac{dz}{dp}(p_0) \cdot \Delta p$$

Prepared Offline

- **Application**: Estimation of the feasible area around one reference trajectory. In green, the first-order corrections that yield a perfect landing, for 10201 initial conditions in horizontal position and speed ($\pm 40\%$ of reference values).




- **Future challenge:** Exploiting symmetries to generate a library with few $x_{ref}$ and $u_{ref}$.

## C

### Vertical optimal-fuel landing with atmosphere

- Here, guidance builds upon the optimal thrust structure of the pure vertical flight.

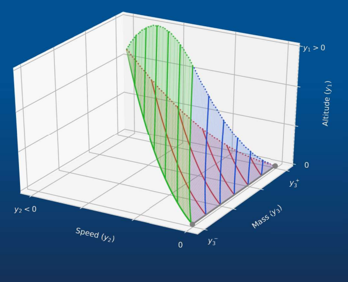
$$\begin{aligned}\text{(Altitude)} \quad & \dot{y}_1 = y_2 \\ \text{(Speed)} \quad & \dot{y}_2 = -g + \frac{g_0 \mathrm{Isp} Q - S_E P_{atm}(y_1)}{y_3} \\ \text{(Mass)} \quad & \dot{y}_3 = -Q\end{aligned}$$

- **Optimal Control Problem**: minimum fuel vertical landing in free final-time.

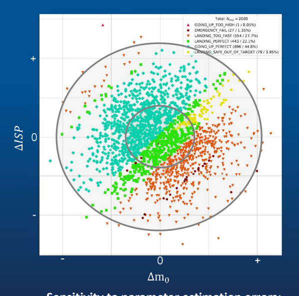
$$\begin{aligned}\max_{Q, t_f} \quad & y_3(t_f) \\ \text{s.t.:} \quad & \dot{y} = f(y) + Qg(y) \\ & y(0) = y^0 \\ & y_1(t_f) = y_2(t_f) = 0 \\ & y_3^- \leq y_3(t) \leq y_3^+, \quad \forall t \in [0, t_f] \\ & Q^- \leq Q(t) \leq Q^+, \quad \forall t \in [0, t_f]\end{aligned}$$

- **Theorem [3]**: The optimal thrust program is **min-max** (one arc may be absent).
- **Safe set** : Vertical flight envelop $\mathcal{F}_{sol}$ using backward flows.

$$\mathcal{F}_{sol} = \left\{ \Phi_{-(f+Q^+g)}\left(\tau^+, \Phi_{-(f+Q^-g)}\left(\tau^-, (0,0,y_3^f)^\top\right)\right) \quad \forall \tau^-, \tau^+ \geq 0 : \ 0 \leq Q^-\tau^- + Q^+\tau^+ \leq y_3^+ - y_3^f, \quad \forall y_3^f \in [y_3^-, y_3^+] \right\}$$




**Safe set**: Vertical flight envelop




**Sensitivity to parameter estimation errors**: Specific Impulse ($\Delta ISP$) and Initial Mass ($\Delta m_0$).

## D References

1. Danylo Malyuta, Taylor P. Reynolds, Michael Szmuk, Thomas Lew, Riccardo Bonalli, Marco Pavone, and Behcet Acikmese. Convex Optimization for Trajectory Generation. arXiv:2106.09125
2. Matthew Clark, Xenofon Koutsoukos, Ratnesh Kumar, Insup Lee, George Pappas, Lee Pike, Joseph Porter, Oleg Sokolsky, "Study on Run Time Assurance for Complex Cyber Physical Systems"
3. Ménou H., Bourgeois E. and Petit N., "Fuel-optimal program for atmospheric vertical powered landing", 60th IEEE Conference on Decision and Control (CDC). IEEE, 2021, *Accepted*.

cnes CENTRE NATIONAL D'ÉTUDES SPATIALES — MINES ParisTech — PSL — ISMME